EuJAP | Vol. 12, No. 1, 2016
UDK
115:512
114/115

ONE SECOND PER SECOND MULTIPLIED BY ONE SECOND*

CLAUDIO MAZZOLA
University of Queensland

ABSTRACT

Detractors of temporal passage often argue that it is meaningless to say that time passes or flows, else time would have to pass at a rate of one second per second, which is in fact not a rate but a number, namely one. Several attempts have been recently made to avoid this conclusion, by retorting that one second per second is in fact not identical to one. This paper shows that this kind of reply is not satisfactory, because it demands a substantive revision of the algebraic behaviour of quantities.

Keywords: *time, flow, rate, speed, quantity*

1.Introduction

Transiency is an undeniable feature of human experience. This fact has led philosophically unprejudiced speakers to coin expressions, such as ‘Time flows’, ‘Time flies’, or ‘Time passes’, which may suggest that time literally and objectively displays a dynamical or flux-like behaviour. Philosophers, however, have long since looked with suspicion at similar figures of speech. To some, these are merely pictorial representations of the way our psychological and physiological constitution affects our subjective experience of temporality; to others, instead, they are rather metaphors of the objectivity of change and becoming.[1] Thinkers from both sides have consequently devised a full battery of arguments to the purpose of establishing once and for all that time does not, nor possibly

[1] Let us observe that, in consequence, not anyone who denies that time literally flows or passes should, for that reason, also deny the objectivity of becoming. Prior (1968) and Tallant (2010), for instance, explicitly declare that the flow of time is only a metaphor, even though they defend the ontological primacy of the present. Similarly, Zeilicovici (1989) proposes a declaredly non-dynamic model of temporal becoming.

*Received: 04.05.2016.
Accepted: 17.12.2016.

could, pass or flow in the literal sense.

The most famous and largely debated argument of this kind is what we may label the *no-rate argument* (Smart 1949, 1954; Price 1996, 2011; Olson 2009; van Inwagen 2009). For short, it can be put as follows:

(1) Everything that flows must flow at some rate or other.

(2) The rate of the flow of time, if any, must be one second per second.

(3) One second per second is identical to one.

(4) One is a number.

(5) Numbers are not rates of flow.

Premises (2)-(5) jointly imply that there is nothing like the rate of the flow of time; thereby, in accordance with (1), it follows that time does not flow.[2]

Those who have traditionally attempted to resist the conclusion of the no-rate argument have typically challenged premises (1) or (2), arguing that time might flow at some meaningful rate of passage other than one second per second (Webb 1960; Zwart 1972, 1976; Schlesinger 1969, 1982; Markosian 1993), or at no meaningful rate at all (Zwart 1972, 1976; Markosian 1993; see also Mazzola 2014). Some of the most recent attacks on the no-rate argument, however, have departed from this tradition, challenging instead premise (3). This new critical trend, initiated by Maudlin (2002, 2007) contends that (a) one second per second is neither identical nor reducible to one, and that (b) accordingly, it is a genuine rate of passage. Premises (2)-(5) consequently fall short of demonstrating that there is nothing like the rate of the flow of time (Phillips 2009; Raven 2010; Skow 2012a).

Let us call the followers of this trend *pro-raters*, and let us collectively refer to theses (a) and (b) as the *pro-rate objection*.[3] This paper is specifically dedicated to show that the pro-rate objection is, at a deeper scrutiny, less appealing than it might seem. More precisely, we shall demonstrate that pro-raters cannot consistently tell what the product of one second per second and one second is equal to, unless they embark on

---

[2] Notice that it is possible to reformulate the argument in semantic terms, so as to lead to the conclusion that it is *meaningless* to say that time flows, or in modal terms, so as to deliver the conclusion that it is *impossible* for time to flow. Such variations, on the other hand, are immaterial to the following discussion. Similarly, we shall not hereafter distinguish between terms such as ‘flow’, ‘pass’, or ‘move’, which are perfectly interchangeable for the sake of the no-rate argument, nor as a consequence between ‘speed’ and ‘rate of passage’.

[3] Not anyone who maintains that time flows at a rate of one second per second will accordingly qualify as a pro-rater in our sense. Once case in point is van Cleve (2011): like Maudlin, he endeavours to establish that one second per second (or, as he says, one hour per hour) is a meaningful rate of passage; however, he does so following in the footsteps of Prior (1958, 1968) and, as a result, he never explicitly addresses (3).

a substantive revision of the algebra of quantities.

2. The pro-rate answer to the multiplication problem

Let us present the pro-raters with the following question: what is one second per second multiplied by one second equal to? Formally put, this question reduces to the following equation

(6) $1 \text{ s/s} \times 1 \text{ s} = x$,

which we shall hereafter label the *multiplication problem*. Pro-raters, we shall argue, cannot offer any consistent solution to this problem, unless they give up some basic and common assumptions concerning the algebraic behaviour of quantities. But how could that be?

Pro-raters claim that one second per second is a genuine rate of passage, so they will presumably interpret the operation on the left-hand side of the equation in (6) as a multiplication between a rate of passage and a temporal duration. Consequently, they will plausibly agree that, as with any other multiplication of that form, the product of the multiplication in (6) should denote a measure of displacement, in adherence to the following schema:

(*) [rate of passage] × [duration] = [displacement].

More specifically, [displacement] should stay for the average distance covered, during a period of time whose duration is specified by [duration], by a mover travelling at the average speed represented by [rate of passage]. To elaborate, this means that a pro-rater should replace the unknown on the right-hand side of (6) with the distance travelled in a unitary interval of time by a mover proceeding at the average speed of one second per second.

To say of something that it literally flows or passes at the constant rate of one second per second, on the other hand, can only mean, if anything, that such thing covers a distance *of one second* per each unit of time elapsed. Therefore, it looks that a pro-rater would be bound to solve (6) in the following way:

(7) $1 \text{ s/s} \times 1 \text{ s} = 1 \text{ s}$.

So far, so good. Problems, however, start showing up when it is recognised that, quite trivially,

(8) $1 \text{ s} = 1 \times 1 \text{ s}$,

and thus (7) must be equivalent to

(9) $1 \text{ s/s} \times 1 \text{ s} = 1 \times 1 \text{ s}$.

Because multiplication is cancellative, this in turn leads to

(10) $1 \text{ s/s} = 1$,

which is precisely what premise (3) asserts, and what pro-raters deny.[4]

This quite simple argument shows that pro-raters can solve the multiplication problem only by renouncing at least one of the auxiliary assumptions respectively leading up to (9) and (10), or else by straightforwardly denying (7). The first alternative, however, would demand renouncing (8), thereby submitting that multiplying a scalar quantity by a number can produce a scalar quantity of a different kind, or maintaining that the multiplication in (9) is not cancellative. The second alternative, instead, would require denying that the product of a unitary rate of passage and a unitary duration be a unitary displacement, thereby violating the schema in (*). For short, this means that pro-raters must choose between remaining silent about the multiplication problem and radically reconceiving the way physical quantities can be algebraically obtained from one another. Either way, the pro-rate objection would lead to a scarcely appealing outcome.[5]

3. Objections

Simple arguments often hide unexpected threats, and there is little doubt that many will look at the above argument with suspicion. The following discussion is meant to dissipate their worries. However, we first need to make some preliminary terminological remarks. To play it safe, we shall borrow our definitions from Skow (2012a), who has arguably offered the most exact and technically informed defence of the pro-rate objection thus far.[6]

---

[4] Let $*$ be an algebraic operation on a given set S. Then, $*$ is said to be cancellative (or to possess the cancellation property) if and only if the following two conditions hold for any a, b, c $\in$ S:

(i) $a * b = a * c \rightarrow b = c$;

(ii) $b * a = c * a \rightarrow b = c$.

[5] Could not the pro-raters deny (7) while keeping (*), say by retorting that the latter schema does not apply to (6), but only to multiplications of the form [rate of passage] × [duration] in which [rate of passage] refers to the speed of material objects travelling in physical space? Or could not they alternatively admit that (6) does indeed satisfy (*), but insist that (7) does not give the correct solution to the multiplication problem? The first line of defence would not do, since (*) is itself but a special case of an even more general schema, according to which the product of a rate of change and a duration is equal to the variation occurred in the dependent quantity of change, whatever it be. Consequently, in that case pro-raters could keep (*) only at the price of contradicting the latter, more general schema. The second line of defence, on the other hand, would put them in the rather uncomfortable position of explaining how anything could move at a rate of one second per second without covering, per each second, a distance of one second. Either way, they would be left in no better predicament than if they chose to simply discard (*).

[6] The only exception is our definition of a numerical value, which Skow leaves implicit. For a more thorough treatment see Suppes and Zinnes (1963).

By a (*positive scalar*) *quantity* we shall mean a property whose determinates can be compared to one another in such a way that their set be isomorphic to the additive semigroup of the (positive) real numbers. This means that such a set is closed under some associative rule of composition, and that some function exists which takes the elements of that set as inputs and gives positive real numbers as outputs, so that the image of the composition of any two determinates in its domain is mapped into the sum of the corresponding images.

Let a *scale* be any such function, let us call the determinates of a quantity its *values*, and let the *numerical values* of a quantity (according to the chosen scale) be the images of its values (according to the scale function). Furthermore, let a scale s be *faithful* just in case there exists a unique value u, such that for any value v in the domain of s the ratio of $s(v)$ to $s(u)$ is identical to the ratio of v to u. Such a value u is what we shall call a *unit* of the given quantity, according to the scale s. Hereafter we shall only consider faithful scales.

Let us suppose, finally, that some class of quantities is taken as fundamental, in the sense that the scales and units employed to measure them suffice to determine the scales and units of all other quantities. Then, we shall say that the class of quantities so chosen uniquely determines a *system of scales*.

Given this conceptual apparatus, let us consider what kinds of objections might be moved to our argument. Because, as we have already noticed, the logical structure of the argument is quite simple, any mistakes we might have made should presumably concern the interpretation of the terms we employed. On the other hand, there can be no doubt as to the meaning of '1', while the referent of '1 s/s' is precisely the matter of contention. Therefore, we ought to question whether we have correctly understood what the pro-raters could mean by '1 s' and '×'.

3.1. Different times

Equation (10) was obtained from (9) thanks to the auxiliary assumption that × is cancellative. Therefore, we have argued, pro-raters can deny the logical inference from (9) to (10) only by denying the latter assumption. Still, it might be objected that this is not necessarily the case. To wit, it may be contended that the quantity denoted by '1 s' on the left-hand side of (7) is not the same quantity as the one that '1 s' denotes on its right-hand side. Therefore, the true reason why (10) does not follow from (9) is that the cancellation property cannot be meaningfully applied to the latter.

Making this objection would indeed make it possible for the pro-raters to deny (10) while maintaining (7), thereby allowing them to offer a simple answer to the multiplication problem without the burden of revising the algebraic behaviour of quantities. The problem with it, however, is that it would make the pro-rate objection entirely irrelevant to the no-rate argument.

Pro-raters, we can safely assume, would presumably consider the quantity on the right-hand side of (7) as a genuine unit of time.[7] Therefore, if they want to insist that the quantity denoted by '1s' on the left-hand side of (7) is a different one, they are evidently obliged to understand it as the unit of some sort of temporal or quasi-temporal quantity other than time itself. The idea of such an additional temporal quantity is actually not new to the debate surrounding the objectivity of temporal passage, and it is equivalently referred to as the *super-time*, or *hyper-time*, or *meta-time*.

Now, why is the idea of the hyper-time detrimental to the pro-rate objection? The reason is that, if it was possible to distinguish between one second of time and one second of hyper-time, then one second of time and one second of hyper-time would have to be different units, and indeed units measuring different quantities. Thus it would be as much appropriate, yet less ambiguous, to refer to the latter unit as one *hyper-second*. This, in turn, would entail that the purported rate of the flow of time should be measured in units of time per unit of hyper-time, and that one second per hyper-second be a different quantity than one second per second, strictly understood as one second of time per second of time.

This fact would have two immediate and related consequences. Firstly, it would falsify premise (2), thus invalidating the no-argument at once.[8] Secondly, and most importantly for the present discussion, proving that one second per second is not identical to one would then establish nothing about the purported rate of the flow of time, which in that case would rather be equal to one second per hyper-second. Either way, rejecting premise (3) would then make absolutely no difference to the no-rate argument, so the pro-rate objection would become entirely moot.[9]

3.2. Different operations

The objection just examined was an attempt to block the logical inference

---

[7] Owing to considerations of symmetry, this assumption will not affect in any way the generality of the following argumentation.

[8] For precisely this reason, the hypothesis of the hyper-time has been sometimes employed as a way to resist the no-rate argument, in particular by Schlesinger (1969, 1982, 1991) and, more recently, by Skow (2012b). For some classical objections to the hyper-time hypothesis see Smart (1949), Williams (1951) and Black (1959).

[9] The same would be true if the two occurrences of '1s' in (7) were respectively claimed to denote, say, one second-of-time-elapsed and one second-of time-covered.

from equation (9) to equation (10). There is, in fact, another way one may try and get to the same result. Rather than distinguishing between different referents of ‘1 s’, one might argue instead that the algebraic operation that appears on the left-hand side of (9) is not of the same kind as the one which appears on its right-hand side: the former one, in fact, holds between two quantities, whereas the latter one holds between a quantity and a number.[10] Once again, this would ensure that the cancellation property does not correctly apply to (9), thus allowing the pro-raters to maintain (7) while denying (10).

The argumentation underlying this type of reply, however, is logically circular. To show why this is so, let us examine it in greater detail. The aim of our hypothetical objectors is to block the logical inference from equation (9) to equation (10). On the other hand, they must subscribe to (9), else they would have to give up (7) or (8), this way exposing their flank to our main argument, and making the current objection worthless as a consequence. Because they hold (9) to be true, then, their objective becomes equivalent to demonstrating that (10) is false.

The argument they set in place to this purpose, as we have seen, is based on the claim that the two algebraic operations that appear in equation (9) are of a different kind, and they are because they hold between different pairs of factors: the former one, in particular, holds between two quantities, whereas the latter one holds between a quantity and a number. On the other hand, because ‘1 s’ is now assumed to have the same meaning on either side, the two operations have one factor in common. Furthermore, that factor is undeniably a quantity, namely a temporal duration. Therefore, the argument underlying the above objection actually reduces to this one: the two algebraic operations in (9) are of a different kind because the non-shared factor on the left-hand side of (9) is a quantity, whereas the non-shared factor on the right-hand side of (9) is a number.

Now, it is evident that hardly anybody would deny the latter clause. This means that, at a deeper analysis, the whole argument rests on the one contention that the non-shared factor on the left-hand side of (9), namely one second per second, is not a number. However, this is clearly but a different way to say that one second per second is not identical to one, which is precisely what the argument under examination was meant to prove. Put in a more condensed form, what our hypothetical objectors argue is that equation (10) is false because the two algebraic operations in (9) are of a different kind, and they are so because (10) is false. This

[10] Some pro-raters, such as Phillips (2009), may object that 1 s/s is not actually a quantity, but rather a relation between quantities (and it is precisely for this reason that 1 s/s cannot be reduced to 1 s / 1s, and hence to 1). This can be easily conceded, since it will make no substantive difference to the argumentation that is about to follow.

argument is evidently circular, and the consequent objection unsound.[11]

3.3. There is no algebra of quantities

So far we have been talking freely about algebraic operations between quantities, or between quantities and numbers. This was admittedly a bit incautious, since the objection that we are about to examine contends precisely that there is in reality nothing like the algebra of quantities, and that the symbol '×', as it is used in equation (7), refers instead to an algebraic operation between numerical values.

To wit, when we compute the average speed of a mover whose displacement in a given duration is known, we do not *literally* divide a length by a duration; rather, we divide the numerical value of the former by the numerical value of the latter, thereby obtaining the numerical value of speed as a result. The fact that the unit of speed is conventionally indicated by 'm/s', therefore, should not erroneously suggest that rates of passage can be obtained by dividing distances by durations, nor that units of speed can be obtained by dividing units of length by units of time. That is rather a mere 'shorthand method of statement', which specifies what unit of speed we ought to adopt if we want to be consistent with the chosen system of scales. However, '[i]t is meaningless to talk of dividing a length by a time', so 'we must not think that we are therefore actually operating with the physical things in any other than a symbolical way' (Bridgman 1922: 29).

Drawing on similar considerations, pro-raters might contend that the algebraic operation that appears in (7) holds in fact between the numerical value of the speed of the flow of time and the numerical value of the unit of duration. Therefore, what (7) actually implies, via (8) and

[11] But, it may be replied, in order to apply the cancellation property to (9) we implicitly presumed that the multiplication signs on either side of the latter referred to the one and the same algebraic operation. Because the operation on the right-hand side of (9) clearly obtains between a number and a quantity, we thereby assumed that the operation on the left-hand side of it should similarly obtain between a number and a quantity, thereby circularly presupposing that 1 s/s be a number. For this reason, one may conclude, our major argument suffers of precisely the same kind of vicious circularity as the one just pointed out. This reply, however, would rest on a false premise. For, while it is certainly true that we assumed that '×' should stay for the same operation on either side of (9), it is not true that, *as a consequence*, such operation should exclusively obtain between numbers and quantities: in fact, nothing *in that assumption* precludes that × could obtain between pairs of quantities *as well as* between quantities and numbers. One such operation could in fact be easily constructed in the way outlined in the next section, modulo some minor modifications. One may certainly retort, at this point, that if the operation in (9) was of a similar kind then it would certainly not be cancellative. However, this remark would hardly point to any circularity, as it would be nothing more than a different way of saying that, under the assumption that × be cancellative, equation (10) logically follows from (9).

(9), is that *the numerical value* of the rate of the flow of time is equal to one. This is, in consequence, all equation (10) entails. Contrary to appearances, (10) is accordingly *not* logically equivalent to (3), so our argumentation is vitiated by equivocation.

Replying to this objection will require a bit of elementary algebra, so to keep things as simple as possible let us agree to identify each quantity with the set of its values. Let thus T be the set of temporal durations and let t be any one of its elements; let s_{T} be the scale chosen to measure durations and let $\mathbf{R}_T$ be its codomain, namely the set of all the possible numerical values $\boldsymbol{r}_T$ that T can take on according to s_T. Similarly, let R be the set whose elements the pro-raters take to be values of the speed of time. Let r be any one of its elements; let s_R be the function that, according to pro-raters, is the scale chosen to measure R, let $\mathbf{R}_R$ be its codomain, and let $\boldsymbol{r}_{\mathrm{R}}$ be any element of $\mathbf{R}_R$.[12]

Let us now briefly recall what algebraic operations are. For the sake of the present discussion, we only need to focus on binary operations. Thus, let A be a non-empty set, and let $A\times A$ be its Cartesian product, namely the set of all possible pairs of elements of A. Then, an algebraic operation on A is simply a map from $A\times A$ to A. The current objection submits that $\times$ is an algebraic operation between the numerical values of the rate of the flow of time and the numerical values of temporal durations, which in particular gives numerical values of temporal durations as a result. This means, therefore, that $\times$ is taken to be a partial function[13] from $(\mathbf{R}_R \cup \mathbf{R}_T)\times(\mathbf{R}_R \cup \mathbf{R}_T)$ to $\mathbf{R}_R \cup \mathbf{R}_T$, where $\mathbf{R}_R \cup \mathbf{R}_T$ is the Boolean union of $\mathbf{R}_R$ and $\mathbf{R}_T$. What the objection denies, on the other hand, is that $\times$ be a partial function from $(R \cup T)\times(R \cup T)$ to $R \cup T$. More generally, the objection has it that no such function as the latter one can possibly be defined.

To rebut that objection, therefore, we shall proceed in two steps. Firstly we shall demonstrate that, as a matter of fact, an algebraic operation from $(R \cup T)\times(R \cup T)$ to $R \cup T$ *can* be meaningfully defined. Secondly, we shall prove that such an operation is in all algebraically identical to $\times$, understood as an operation between numerical values.

So, let us concede that $\times$ be a partial function from $(\mathbf{R}_R \cup \mathbf{R}_T)\times(\mathbf{R}_R \cup \mathbf{R}_T)$ to $\mathbf{R}_R \cup \mathbf{R}_T$, as the objection wants it to be. This means that $\times$ takes ordered pairs of the form $(\boldsymbol{r}_R, \boldsymbol{r}_T)$ as the input, and it gives some numerical

---

[12] Notice that, if premises (2)-(4) are jointly true, then R will be not a set of values, but a set of numbers. By the same token, in that case s_T will be a ratio-preserving function from numbers to numbers. For ease of exposition, we shall hereafter keep similar parenthetical remarks implicit.

[13] The reason why $\times$ is a partial function is that it is restricted to ordered pairs of the form $(\boldsymbol{r}_R, \boldsymbol{r}_T)$ or $(\boldsymbol{r}_T, \boldsymbol{r}_R)$, to the effect that its domain does not include any ordered pair of the form $(\boldsymbol{r}_R, \boldsymbol{r}_R)$ or $(\boldsymbol{r}_T, \boldsymbol{r}_T)$. Notice, further, that while the codomain of $\times$ is $\mathbf{R}_R \cup \mathbf{R}_T$, its image coincides with $\mathbf{R}_T$.

value $\times(\boldsymbol{r}_R, \boldsymbol{r}_T)$ as the output. Furthermore let us notice that, by definition, a faithful scale must preserve the ratios between its arguments, to the effect that the ratio between any two numerical values in its codomain must be equal to the ratio between the corresponding counterimages. From this, it straightforwardly follows that a faithful scale must be injective, i.e. that to each numerical value in $\mathbf{R}_R$ corresponds exactly one value of speed, and to each numerical value in $\mathbf{R}_T$ corresponds exactly one value of duration. This also guarantees that the inverse functions s_R^{-1} and s_T^{-1} of the scales s_R and s_T exist. So, given these basic ingredients, here is the recipe to construct our map.

First of all, take some ordered pair of the form $(\boldsymbol{r}_R, \boldsymbol{r}_T)$ from $(\mathbf{R}_R \cup \mathbf{R}_T)\times(\mathbf{R}_R \cup \mathbf{R}_T)$. Next, apply two different projections to $(\boldsymbol{r}_R, \boldsymbol{r}_T)$, thereby obtaining $\boldsymbol{r}_R$ and $\boldsymbol{r}_T$ as a result. Then, for each such numerical value, determine the value to which the latter is assigned by means of the chosen scale: as we have just pointed out, this value must exist and it is unique. Take the two values $s_R^{-1}(\boldsymbol{r}_R)$ and $s_T^{-1}(\boldsymbol{r}_T)$ so obtained and combine them so as to form the ordered pair $(s_R^{-1}(\boldsymbol{r}_R), s_T^{-1}(\boldsymbol{r}_T))$, whose first term is a value of speed and whose second term is a value of duration.

In the meanwhile apply $\times$ to $(\boldsymbol{r}_R, \boldsymbol{r}_T)$. Take the numerical value $\times(\boldsymbol{r}_R, \boldsymbol{r}_T)$ so obtained and determine its counterimage as of the chosen scale for temporal durations, thus getting $s_T^{-1}(\times(\boldsymbol{r}_R, \boldsymbol{r}_T))$. Once again, the existence and uniqueness of this value are guaranteed by the faithfulness of s_T. Finally, take $s_T^{-1}(\times(\boldsymbol{r}_R, \boldsymbol{r}_T))$ along with the ordered pair already in your possession, so as to generate the ordered pair $((s_R^{-1}(\boldsymbol{r}_R), s_T^{-1}(\boldsymbol{r}_T)), s_T^{-1}(\times(\boldsymbol{r}_R, \boldsymbol{r}_T)))$.

Repeat the whole procedure for all ordered pairs consisting of a numerical value of speed and a numerical value of duration (and vice-versa), and gather the ordered pairs so obtained in one set. Let us call it $\otimes$. It is then immediate to see that $\otimes$ is a partial function from $(R \cup T)\times(R \cup T)$ to $R \cup T$, exactly as desired. This should suffice to prove that an algebraic operation between quantities such as speed and duration can be meaningfully defined.

Let us accordingly move on and let us show, as promised, that $\otimes$ is algebraically equivalent to $\times$. This can be demonstrated quite easily. Let f be the union of s_R and s_T. This means that f is a function from $R \cup T$ to $\mathbf{R}_R \cup \mathbf{R}_T$ such that, for any element x of $R \cup T$, $f(x) = s_R(x)$ if x is a value of speed, whereas $f(x) = s_T(x)$ if x is a value of duration. Notice that, because s_R and s_T are injective, so must be f, so the inverse function f^{-1} of f is well-defined. Now, take any pair of values in the domain of $\otimes$. Because of the very definition of $\otimes$, there must be $\boldsymbol{r}_R$ and $\boldsymbol{r}_T$ such that the pair just chosen must be unambiguously expressible as $(s_R^{-1}(\boldsymbol{r}_R), s_T^{-1}(\boldsymbol{r}_T))$. Then it is elementary to check that, by virtue of the very construction of $\otimes$, the following must be true:

(11) $\otimes(s_R^{-1}(\boldsymbol{r}_R), s_T^{-1}(\boldsymbol{r}_T)) = s_T^{-1}(\times(\boldsymbol{r}_R, \boldsymbol{r}_T))$.

This, on the other hand, is but a different restatement of:

(12) $s_R^{-1}(\boldsymbol{r}_R) \otimes s_T^{-1}(\boldsymbol{r}_T) = s_T^{-1}(\boldsymbol{r}_R \times \boldsymbol{r}_T)$.

Thanks to the definition of f, we thereby get:

(13) $f^{-1}(\boldsymbol{r}_R) \otimes f^{-1}(\boldsymbol{r}_T) = f^{-1}(\boldsymbol{r}_R \times \boldsymbol{r}_T)$.

Because $(s_R^{-1}(\boldsymbol{r}_R), s_T^{-1}(\boldsymbol{r}_T))$ was arbitrarily chosen, this is enough to show that f^{-1} is a partial magma homomorphism between $(\mathbf{R}_R \cup \mathbf{R}_T, \times)$ and $(R \cup T, \otimes)$; furthermore, it would be elementary to show that f^{-1} preserves cancellativity. This means, in particular, that if × satisfies the ordinary rules of multiplication that we employed in order to derive (10) from (7), then so must do ⊗. This proves that, however one chooses to interpret '×', it is always possible to restate our main argument in terms of an algebraically equivalent operation ⊗, which does not hold between (numbers and) numerical values, but between (numbers and) quantities. Therefore, our argument suffers from no equivocation.

Before claiming victory, however, a possible counter-reply is worth a brief mention. To carry out our construction of ⊗, we took it for granted that operations such as Boolean unions and Cartesian products can be meaningfully defined on sets of values (and numbers). However, one might contend, this is precisely what the objection under examination denies: according to it, quantities are 'physical things', and as a consequence they cannot undergo the same sort of logico-mathematical manipulations as abstract entities such as sets and numbers. Therefore, our entire discussion is vitiated by a *petitio principii*.

This further worry, however, is easily dispelled. The whole construction of ⊗, as it can be easily checked, was directly based on the definitions given at the beginning of § 3, and it presupposed nothing about quantities which was not already taken for granted by those definitions. Just to make but one example, the very definition of a scale assumes that it is possible to take the Cartesian product of a set of values and a set of numbers. For consistency, anyone who wishes to make the above reply will then have to reject our definitions. The burden will be up to them, however, of proving that the conceptual foundations of measurement theory can be laid down without ever mentioning sets of values, or functions from values to numbers.

4. Conclusion

The no-rate argument is certainly one the strongest philosophical challenges to the idea that time possesses objective dynamical or flux-like properties. Not by chance, throughout the last sixty years, philosophers who believe in the objectivity of the flow of time have made numerous attempts to avoid its conclusion. Pro-raters, in particular, insist that time may flow indeed at a rate of one second per second, because one second per second is not identical to one.

Even though the pro-rate objection has received much support in recent years, this paper has shown that it is in fact less appealing than it might look. In fact, we have demonstrated that pro-raters cannot consistently calculate the distance covered by time in a temporal unit, unless they want to insist that quantities do not satisfy the standard rules of multiplication. This result, of course, by no means guarantees that the no-rate argument is safe from rebuttal. Nonetheless, it certainly raises the question whether its rejection is worth the cost of a radical revision of the algebra of quantities.

REFERENCES

Black, M. 1959. The direction of time. *Analysis* 19: 54-63.

Bridgman, P. W. 1922. *Dimensional Analysis*. New Haven: Yale University Press.

Markosian, N. 1993. How fast does time pass? *Philosophy and Phenomenological Research* 53: 829-844.

Maudlin, T. 2002. Remarks on the Passing of Time. *Proceedings of the Aristotelian Society* 102: 259-274.

Maudlin, T. 2007. *The Metaphysics within Physics*. Oxford: Oxford University Press.

Mazzola, C. 2014. Does time flow, at any rate? *Metaphysica* 15: 157-172.

Olson, E. T. 2009. The rate of time's passage. *Analysis* 69: 3-9.

Phillips, I. 2009. Rate abuse: A Reply to Olson'. *Analysis* 69: 503-505.

Price, H. 1996. *Time's Arrow and Archimedes' Point*. Oxford: Oxford University Press.

Price, H. 2011. The Flow of Time. In *The Oxford Handbook of Philosophy of Time*, ed. C. Callender, 276-311. Oxford: Oxford University Press.

Prior, A. N. 1958. Time after time. *Mind* 67: 244-246.

Prior, A. N. 1968. Changes in Events and Changes in Things. In *Papers on Time and Tense*, ed. A. N. Prior, 1-14. Oxford: Oxford University Press.

Raven, M. J. 2010. Can time pass at the rate of 1 second per second? *Australasian Journal of Philosophy* 88: 1-7.

Schlesinger, G. N. 1969. The two notions of the passage of time. *Noûs* 3: 1-16.

Schlesinger, G. N. 1982. How time flies. *Mind* 91: 501-523.

Schlesinger, G. N. 1991. E pur si muove. *The Philosophical Quarterly* 41: 427-441.

Skow, B. 2011. On the meaning of the question “How fast does time pass?”. *Philosophical Studies* 155: 325-344.

Skow, B. 2012a. One second per second. *Philosophy and Phenomenological Research* 85: 377-389.

Skow, B. 2012b. Why does time pass? *Noûs* 46: 223-242.

Smart, J. J. C. 1949. The river of time. *Mind* 58: 483-472.

Smart, J. J. C. 1954. The temporal asymmetry of the world. *Analysis* 14: 79-83

Suppes, P. and J. L. Zinnes. 1963. Basic Measurement Theory. In *Handbook of Mathematical Psychology*, ed. R. D. Luce, R. R. Bush and E. Galanter, 1-76. New York: Wiley.

Tallant, J. 2010. A sketch of a presentist theory of passage. *Erkenntnis* 73: 133-140.

van Cleve, J. 2011. Rates of passage. *Analytic Philosophy* 52: 141-170.

van Inwagen, P. 2009. *Metaphysics*, third edition. Boulder: Westview Press.

Webb, C. W. 1960. Could time flow? If so, how fast? *The Journal of Philosophy* 57: 357-365.

Williams, J. 1951. The myth of passage. *The Journal of Philosophy* 48: 457-472.

Zeilicovici, D. 1989. Temporal becoming minus the moving-now. *Noûs* 23: 505-524.

Zwart, P. J. 1972. The flow of time. *Synthese* 24: 133-158.

Zwart, P. J. 1976. *About Time*. Amsterdam and Oxford: North-Holland.